Experiment #: 16

Experiment Title: Wheatstone bridge

Objectives:

1. To learn how to measure the coefficient of resistance of different metal wires using a

Wheatstone bridge

Theory:

The Wheatstone bridge is a circuit used to compare an unknown resistance with a known resistance. The bridge is commonly used in control circuits. For instance, a temperature sensor in an oven often consists of a resistor with a resistance that increases with temperature. This temperature-dependent resistor is compared with a control resistor (outside the oven) to control a heater and maintain a set temperature. A schematic of a Wheatstone bridge is shown below:

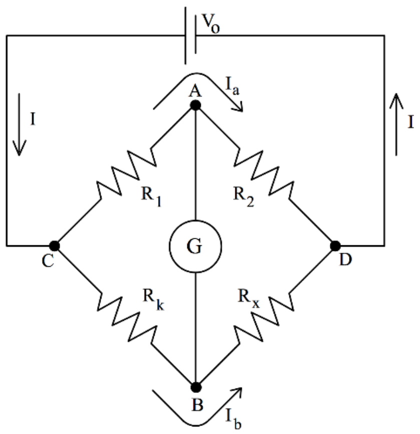


Fig.1. Schematic of a Wheatstone Bridge

The unknown resistor is R_x, the resistor R_k is known, and the two resistors R_1 and R_2 have a known ratio R_2 R_1 , although

their individual values may not be known. A galvanometer G measures the voltage difference V_{AB} between points A and B. Either the known resistor R_k or the ratio R_2 R_1 is adjusted until the voltage difference V_{AB} is zero and no current flows through G. When V_{AB} = 0, the bridge is said to be “balanced”. The unknown resistor is R_x, the resistor R_k is known, and the two resistors R_1 and R_2 have a known ratio R_2 R_1 , although their individual values may not be known. A galvanometer G measures the voltage difference V_{AB} between points A and B. Either the known resistor R_k or the ratio R_2 R_1 is adjusted until the voltage difference V_{AB} is zero and no current flows through G. When V_{AB} = 0, the bridge is said to be “balanced”.

I_b Since $V_{AB} = 0$, the voltage drop from C to A must equal the voltage drop from C to B, $V_{CA} = V_{CB}$. Likewise, we must have $V_{AD} = V_{BD}$. So we can write,

(1) $$I_a R_1 = I_b R_k$$

(2) $$I_a R_2 = I_b R_x .$$

Dividing (2) by (1), we have

(3) $$\frac{R_2}{R_1} = \frac{R_x}{R_k} \quad , \quad R_x = R_k \frac{R_2}{R_1} .$$

Thus, the unknown resistance R_x can be computed from the known resistance R_k and the known ratio R_2 R_1 . Notice that the computed R_x does not depend on the voltage V_o; hence, V_o does not have to be very stable or well-known. Another advantage of the Wheatstone bridge is that, because it uses a *null measurement*, ($V_{AB} = 0$), the galvanometer does not have to be calibrated.

In practice, the Wheatstone bridge is seldom used merely to determine the value of a resistor in the manner just described. Instead, it is usually used to measure small changes in R_x due, for instance, to temperature changes or the motion of microscopic defects in the resistor. As an example, suppose $R_x = 10^6$ Ω and we wanted to measured a change in R_x of 1Ω, resulting from a small temperature change. There is no ohmmeter which can reliably measure a change in resistance of 1 part in a million. However, the bridge can be set up so that $V_{AB} = 0$ when R_x is exactly 10^6 Ω. Then any change in R_x , ΔR_x ,would result in a non-zero V_{AB}, which, as we show below, is proportional to ΔR_x . You would not weigh a cat by weighing a boat with and without a cat on board. Likewise, you would not want to measure very small changes in R_x by measuring R_x with and without the change. Instead, you want to arrange things so that the change in R_x,

ΔR_x , is the entire signal. The bridge serves to “balance out” the signal due R_x, leaving only the signal due to ΔR_x . To show that $V_{AB} \propto \Delta R_x$,we consider Fig.1, and note that $V_{CD} = V_o$. We assume that the galvanometer is a perfect voltmeter, so that no current flows through it, even when the bridge is not balanced. We also assume that the bridge has been balanced with the sample resistance at an initial value of R_{xo} , so that $R_{xo} = R_k(R_2\ R_1)$. Then we consider what happens to V_{AB} when the sample resistor is changed by a small amount to a new value $R_x = R_{xo} + \Delta R_x$. Applying Kirchhoff’s Voltage Law and Ohm’s Law to the upper and lower arms of the bridge, we have

$$V_o = I_a(R_1 + R_2) = I_b(R_k + R_x)..$$

We are trying to find V_{AB}, which we can relate to I_a and I_b.

$$V_{AB} = V_A - V_B = (V_C - V_B) - (V_C - V_A) = I_b R_k - I_a R_1.$$

We can use (4) to solve for I_a and I_b and then substitute for I_a and I_b in (5),

$$V_{AB} \quad = \quad V_o \frac{R_k}{R_k + R_x} - V_o \frac{R_1}{R_1 + R_2}.$$

This equation shows how V_{AB} depends on R_x. Notice that eqn.(6) yields V_{AB} = 0 when R_x = R_k (R_2 R_1). To see how much V_{AB} changes when R_x changes from R_{xo} to R_{xo} +ΔR_x,wewrite

$$\Delta V_{AB} \quad \cong \quad \left(\frac{dV_{AB}}{dR_x}\right)_{R_x = R_{xo}} \cdot \Delta R_x \quad = \quad -V_o \frac{R_k}{(R_k + R_{xo})^2} \cdot \Delta R_x.$$

We wrote eqn.(7) by regarding V_{AB} as a function of R_x, and remembering (from Calculus) that if f = f(x), then $\delta f = \frac{df}{dx}\delta x$. Substituting R_{xo} = R_k(R_2 R_1) into (7) yields

$$\Delta V_{AB} \quad = \quad \frac{-V_o\ R_k}{\left(R_k + R_k \frac{R_2}{R_1}\right)^2} \cdot \Delta R_x \quad = \quad \frac{-V_o}{R_k\left(1 + \frac{R_2}{R_1}\right)^2} \cdot \Delta R_x \quad = \quad \frac{-V_o\ R_1^2}{R_k(R_1 + R_2)^2} \cdot \Delta R_x.$$

Finally, we remember that $V_{AB,initial}$ = 0, so the change in V_{AB} is the same as V_{AB}, and we have

$$V_{AB} \quad = \quad V_o\left(\frac{R_1}{R_1 + R_2}\right)^2 \frac{\Delta R_x}{R_k}.$$

We are done. We have shown that, when the bridge is balanced, any small change in R_x will produce a V_{AB} proportional to that change.

We will use a slide-wire Wheatstone bridge, in which the two resistors R_1 and R_2 are two portions of a single, uniform Ni-Cr wire. Electrical contact is made at some point along the wire by a sliding contact (this contact corresponds to point A). The two portions of the wire on either side of the contact have resistances R_1 and R_2, and the ratio R_2 R_1 is the same as the ratio of the lengths of the two portions of wire, L_2 L_1 . The lengths are readily measured with a meter stick which the wire rests upon. A digital multimeter (DMM) in voltage mode will serve as the galvanometer.

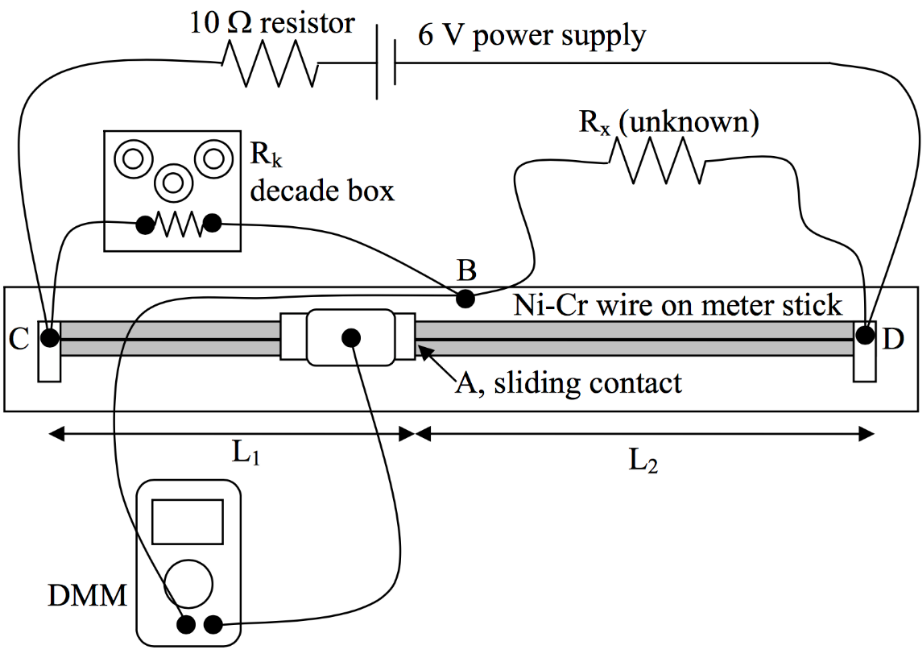
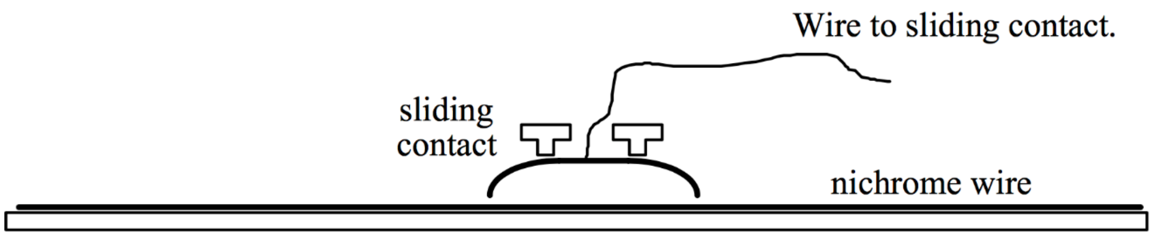


Fig. 2. Physical Layout of the Wheatstone Bridge.

The 10Ω resistor in series with an adjustable power supply serves to limit the current through the bridge to less than 1A. (Higher currents might over-heat components of the bridge.) Set the voltage from the power supply to approximately 6 volts with the “Coarse” adjust voltage knob.

The known resistor R_K is adjustable and can be set to any value from 1Ω to 999Ω in 1Ω steps with a decade resistance box, which is accurate to 0.02%. The Ni-Cr wire has a total resistance of about 2Ω. The sliding contact is spring loaded; you have to push it down to make contact to the wire at point A. There are two buttons on the sliding contact; push one or the other, but not both, to contact the wire.



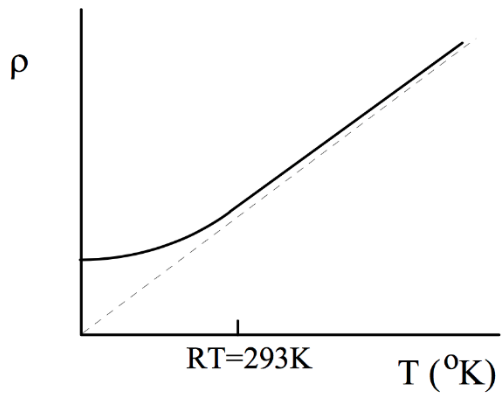
Adjust the position of the sliding contact to balance the bridge (zero reading on the DMM).

ForPart1ofthelab,theunknownresistorR_X isoneof5coilsofwire,numbered 1 through 5, mounted on a board. The lengths of the wires, their composition, and their *gauge number* are printed on the board. The gauge number is a measure of the thickness of the wire. 22 gauge wire has a diameter of 0.644 mm; 28 gauge wire has a diameter of 0.321 mm. For part 2 of the lab, R_X is a stand-alone coil of copper.

The resistance R of a wire is related to its length L, its cross-sectional area A, and its *resistivity* ρ by the formula

$$R = \rho \frac{L}{A}.$$

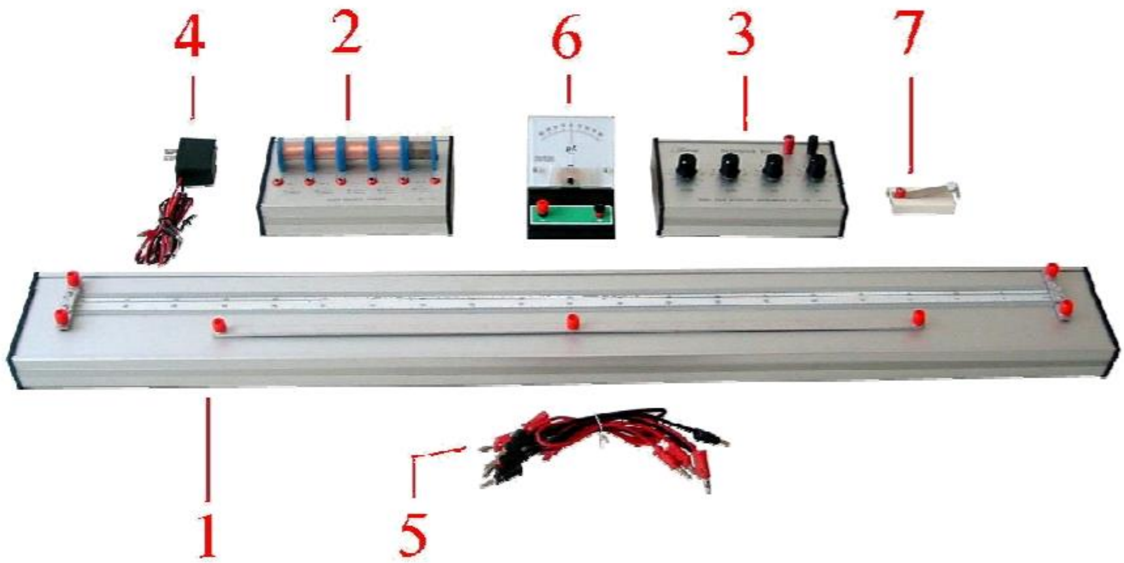
The resistivity ρ of a material depends on composition, on defect structure, and on the temperature. For metals, ρ is approximately constant at very low temperatures (T < 100K) and increases approximately linearly with temperature (measured in K) at high temperatures.



At T = 20C, the resistivity of pure, defect-free, copper is $\rho_{RT} = 1.678 \times 10^{-8}\,\Omega \cdot \mathrm{m}$. (RT stands for room temperature.)

List of Equipment:

Apparatus



NO	Accessory	Quantity
1	Slide Wire Bridge	1
2	Unknown Resistance Box	1
3	Variable Resistance Box	1
4	DC Power Supply	1
5	Wire	6
6	Galvanometer	1
7	Probe	1

Circuit Diagram:

Procedure:

Part 1: Resistivity of Copper

In this section, you will use the bridge to make precise measurements of the resistance of each of the 5 coils of wire on the coil board. First, however, use the digital multimeter (DMM) to make an approximate measurement of the resistance of each coil. Also use the DMM to see how the resistance of the decade box changes when you turn the knobs. You must temporarily disconnect the coil board and the decade box from the bridge when testing them with the DMM. With the DMM, measure the resistance of each of the five coils to the nearest 0.1 Ω. The resistance of the wire leads used to connect the DMM to the coils is not negligible, so first use the DMM to measure the resistance of the two wire leads in series, and then subtract this lead resistance from your measurements. *Check yourself:* the smallest coil resistance is below 1 ohm!

Now familiarize yourself with the decade resistance box. Adjust the knobs for 15 ohms and verify with the DMM that the value is indeed 15 ohms plus the resistance of
the wire leads. For this measurement, disconnect the decade box from the bridge circuit and connect the DMM directly to the decade box.

Now use the bridge to measure the resistance of each of the 5 coils. Before connecting the power supply to the bridge, carefully check that all the connections are correct. Select a coil, attach it to the bridge, and set the decade box resistance R_k to be as near to R_x as possible (you know R_x roughly from your DMM measurements). Balance the bridge by moving the sliding contact along the wire while watching the DMM. Remember that when using the galvanometer you should always begin with button 1, then button 2, and finally button 3. With the bridge balanced, measure L_1 and L_2, and compute

$$R_x = R_k \frac{R_2}{R_1} = R_k \frac{L_2}{L_1} \ .$$

Repeat this procedure for the other 4 coils.

For each of these coils, compute the resistivity, using your measured resistances, the data printed on the coil board, and eq'n (10). Make a table with the headings: coil #, R from DMM, R from bridge, and computed resistivity. Four of the 5 sample coils are made of copper (Cu). For these four coils, compute the average resistivity, and the uncertainty of the average (σ_{mean}). Compare your average value with the known value.

Coil #5 is made of a copper-nickel alloy. What is the ratio ρ_{Cu-Ni} ρ_{Cu} ?

Part 2. Temperature dependence of resistivity.

For this part, the unknown R_X is a copper wire coil (the one which is not attached to the coil board). We will use eq'n(9) to measure the change in resistance of the coil when its temperature is changed by plunging it into an ice bath. Note that eq'n(9) can be written as

$$V_{AB} \quad = \quad V_o \left(\frac{L_1}{L_1 + L_2} \right)^2 \frac{\Delta R_x}{R_k}$$

Prepare an ice bath by filling a beaker with ice from the freezer (ask your TA where) and adding some water. Measure room temperature and the ice bath temperature with the digital thermometer.

Measure the coil resistance R_X with the DMM and set R_K equal to R_X, as nearly as possible. With R_X and R_K connected in the bridge, measure $V_o = V_{CD}$ with the DMM.

See Fig. 2 to locate V_{CD}. Note that V_{CD} is not the power supply voltage. For this bridge measurement, we must replace the galvanometer in the bridge circuit with the DMM, set to DC volts with the most sensitive range(400mV). We need to do this because the galvanometer does not behave as an ideal voltmeter, as assumed in the derivation of eq'n (9).

With the sample coil at room temperature, balance the bridge and compute the coil's resistance at room temperature. Now without changing the position of the slide wire contact, place the coil into the ice bath and watch as the voltage V_{AB} on the DMM changes. Record V_{AB} after it reaches equilibrium. Use eq'n(9) to compute ΔR_X , the change

In resistance of the coil. As a measure of how sensitive the resistivity is to temperature changes, we can compute the fractional change in the resistivity divided by the change in temperature,

$$\frac{(\Delta\rho/\rho)}{\Delta T} = \frac{1}{\rho}\frac{\Delta\rho}{\Delta T} = \frac{1}{R}\frac{\Delta R}{\Delta T}.$$

This quantity, multiplied by 100%, is the % change per degree. From your measurements, compute the % change in resistivity per degree for copper, and compare this value with the value you expect if $\rho \propto T(^{O}K)$. [Hint: If $\rho \propto T(^{O}K)$,then
$\rho = C \cdot T$,whereCissomeconstant,and $\Delta\rho = C \cdot \Delta T$.]

Questions

1. Two copper wires, labeled A and B, are at the same temperature, but the temperature is unknown. Wire B is three times as long and has 1/3 the diameter of wire A. Compute the ratio R_B/R_A.

2. A 28 gauge copper wire is 10 meters long. What is its resistance at room temperature? **3.** Name two advantages of a Wheatstone bridge over an ordinary ohmmeter.

4. What is the formula for the total resistance of two resistors in parallel? Consider two resistors $R_1 = 2\ \Omega$ and $R_2 = 200\ \Omega$. What is the total resistance of these two resistors in parallel. Give your answer to the nearest ohm.

5. Consider the circuit shown in Fig.2 (not Fig.1) and assume that the galvanometer acts like an ideal voltmeter (with infinite internal resistance). Recall that the total resistance of the Ni-Cr wire is 2Ω. If R_K and R_X are both very, very large compared to 2Ω, how much current flows through the Ni-Cr wire?

6. Suppose an unknown resistance R_X is measured with the bridge circuit shown in Fig. 2 and the result is $R_X = 8.65\Omega$. The 6V power supply is then replaced with an 10V supply and R_X is re-measured. What is the new measured value of R_X?

7. A sample of copper wire is 200m long and has a diameter of 0.150 mm. Its resistance is determined to be 88.0Ω. What is the resistivity of the copper in this wire?

8. (Counts as 3 questions) A Wheatstone Bridge such as in Fig. 1 has a $V_O = 10.0V$. With $R_K = 5.00\Omega$ and the unknown resistor R_X at room temperature, the bridge is balanced with $L_1 = 34.0cm$ and $L_2 = 66.0$ cm. What is the value of R_X? While still attached to the bridge, the unknown R_X is placed in an oven, which raises its temperature by 33.0^{O} C. The value of

V_{AB} is then found to be 0.127V. What is the resistance change of the unknown, ΔR_X? What is the % change per degree of the resistivity of this sample?

Data Collection:

Unknown resistance	Resistance $R_1 (\Omega)$	$\overline{MB}$ (cm)	$\overline{BN}$ (cm)	Unknown resistance			$\rho (\Omega \cdot m)$
				$R_2 (\Omega)$ $R_2 = R_1(\frac{\overline{BN}}{\overline{MB}})$	D (m)	L (m)	$\rho = \frac{\pi D^2 R_2}{4L}$
a							
a							
a							

Calculation:

MN is the metal wire, the point B divides MN into MB and BN , and the resistances are R3 And R4 Each resistance is proportional to its length, so

$$R = \rho \frac{L}{A}$$

 is the resistivity, L is the wire length, and A is the wire cross-sectional area. So the ratio of R3 and R4 is equal to the ratio of MB and BN . If we can find a spot to make the reading become zero, and equation (5) can be written as:

$$R_2 = R_1(\frac{\overline{BN}}{\overline{MB}})$$

Result: